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Irwin 12266

IRM 0BP

College Entrance Examination Board
Advanced Placement Examination
MATHEMATICS
SECTION II

Time—1 hour and 30 minutes

1. Determine the following about the graph of the equation

$$y = \frac{8}{x^3} - \frac{6}{x}$$

- (a) Is the graph symmetric with respect to

- (1) the X-axis?
(2) the Y-axis?
(3) the origin?

- (b) Identify all asymptotes of the graph.

- (c) Find the x-coordinate of each point at which y is a relative maximum and of each point at which y is a relative minimum. Justify your answer.

- (d) Find the x-coordinate of each point of inflection. Justify your answer.

2. A right circular cone has base radius 5 and altitude 12. A cylinder is to be inscribed in the cone so that the axis of the cylinder coincides with the axis of the cone. Given that the radius of the cylinder must be between 2 and 4 inclusive, find the value of that radius for which the lateral surface area of the cylinder is minimum. Justify your answer. (Note: The lateral surface of a cylinder does NOT include the bases.)

3. Find the area of the region in the first quadrant enclosed by the graph of $y = \ln x$, the line tangent to $y = \ln x$ at $x = 3$, and the coordinate axes. (Note: $\ln x$ denotes the logarithm of x to the base e .)

4. A solid has an elliptical base with major axis 6 units and minor axis 3 units. Each cross section perpendicular to the major axis is a square. One side of the square is a chord of the ellipse parallel to the minor axis. Find the volume of this solid.

5. Determine and evaluate a definite integral for which $0.1\left(\frac{1}{2}\sqrt{4.0} + \sqrt{4.1} + \dots + \sqrt{8.9} + \frac{1}{2}\sqrt{9.0}\right)$ is a trapezoidal approximation. Which is greater, the value of the integral or the trapezoidal approximation? Why?

6. Parametric equations of motion of a particle are $\begin{cases} x = \sec \pi t \\ y = \tan \pi t \end{cases}$, $0 \leq t < \frac{1}{2}$, where t denotes time.

- (a) Find the x and y components of the velocity vector at $t = \frac{1}{4}$, and determine the speed of the particle at $t = \frac{1}{4}$.
(b) By eliminating t , find a Cartesian equation for the conic section on which the particle moves.
(c) Identify the type of conic, and indicate by a sketch the path of the particle for the given time interval.

7. Prove:

(a) $1 + x \leq e^x$ for all real x .

(b) $e^x \leq \frac{1}{1-x}$ for all $x < 1$.

END OF EXAMINATION

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